考試時間	月 日上午 下午第 節 (星期) 晚間	份數		任課教師	

國立臺灣科技大學 108學年度第 2 學期 考試命題用紙 第 1 頁共 1 頁

考試科目：Algorithms

☑研究所
☐大學部
☐工程在職進修

系班別：博七班資格考

1. (30%) Among all the sorting algorithms, quick sort and merge sort are two representatives. In order to avoid the worst-case behavior for classic quick sort algorithm, the randomized quick sort algorithm is a variant.

   (a) Please calculate the expected running time of the randomized quick sort algorithm.

   (b) Please write down the pseudo codes for the randomized quick sort and the merge sort algorithms, respectively.

   (c) Please give an example to show that the merge sort algorithm could be a better choice than the classic quick sort algorithm.

2. (12%) For a given ordered number sequence "50, 1, 3, 5, 25, 2, 4, 6, and 7," please write down all the steps for building:

   (a) A binary search tree

   (b) A B-tree

   (c) An AVL-tree

3. (8%) Suppose the complexity of a recursive function $\mathrm{T}(n) = 2\times\mathrm{T}(\lfloor\frac{n}{2}\rfloor)+n$ is $\mathrm{O}(n\log_2 n)$, please calculate the complexity for $\mathrm{T}(n) = 2\times\mathrm{T}(\lfloor\sqrt{n}\rfloor) + \log_2 n$.

4. (15%) Rank the following functions according to ascending growth rate: $\log n$, $n^{\log\log n}$, $\log n!$, $2^{\sqrt{2\log n}}$, $\sqrt{\log n}$.

5. (a) (15%) A subsequence of a sequence is a sequence that appears in the same relative order, but not necessarily contiguous. Let S_1 and S_2 be two sequences. Give an algorithm in detailed steps that produce a shortest sequence S containing both S_1 and S_2 as its subsequences.

   (b) (5%) Give an example to demonstrate how your algorithm works.

6. (15%) **The Knapsack Problem**: Given a set S of n items, where each item i is with profit p_i and size s_i, and a knapsack with capacity B (of course $B \geq s_i$, $\forall i$). Find a subset (meaning, no fractional selections) of items whose total size is bounded by B and the total profit is maximized.

   A basic greedy strategy for this is to sort the items by decreasing profit-to-size ratio, then pick items in that order until the knapsack is filled.

   Prove that this basic greedy strategy is not only suboptimal, but it can perform arbitrarily badly.